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THE IMPACT OF THE CALCULATOR ON THE CONTENT VALIDITY OF ADVANCED PLACEMENT CALCULUS PROBLEMS

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ABSTRACT

Nineteen Calculus II students were randomly sampled and divided into calculator (n = 9) and noncalculator groups (n = 10). These students were asked to "think aloud" while solving 24 Advanced Placement calculus problems. Each student interview was videotaped, coded and analyzed for reasoning process as well as outcome. The results indicated that the use of calculators affected the testing of basic facts and reasoning processes at the individual problem level but basic concepts and principles were unaffected. A follow-up teaching experiment suggested that the content validity of Advanced Placement calculus problems could be more seriously affected if students had greater calculator expertise.

Introduction

The College Board Mathematics Advisory Committee has announced that beginning January 1983, the use of calculators will be permitted, but not required, on the AP calculus examination. Such a policy change is appropriate due to the large number of students who now own calculators and the ever increasing number of teachers who allow students to use calculators in the classroom. It is also supportive of such major professional organizations as NCTM which encourages mathematics programs to take full advantage of calculators and computers at all educational levels.

The challenge to us as mathematics educators is to take full advantage of the power of calculators without sacrificing other goals of instruction. Most mathematics educators maintain that certain basic facts, concepts, principles and reasoning processes are important for students to learn. However, in some cases the calculator makes it possible for students to reach correct answers by using the calculator's circuitry rather than the intended method of solution. Consequently, important questions come to mind: (1) At the current level of calculator expertise among students, how would the content validity of the individual AP calculus problems be affected if the calculator were allowed during testing? and (2) What is the potential of the calculator to change the content validity of the individual AP calculus problems?

In 1980 a small research study was conducted at MTU for the purpose of obtaining an answer to the first question, then calculator calculus materials were reviewed and a teaching experiment was conducted for the purpose of conjecturing answers to the second question.

The Research Study

Method of Sampling

Nineteen MTU students were randomly sampled from the Calculus II rosters during the final exam week of spring and summer quarters, 1980. Students who were chosen to participate in the study were given a letter briefly describing the study and requesting their participation. Each letter was followed by a telephone call to obtain the student's response. If the student was willing to participate in the study, an interview time was arranged. Nine students chose to work the calculus test with their calculator (the experimental group) and the remaining ten students worked the test without their calculators (the control group). A comparison of Calculus I and II grade point averages between the two groups showed no significant difference ($p < .05$) between their achievement levels. Each student was given $3 credit at the campus bookstore for participating in the study.

Method of Data Collection

Permission was granted by ETS to use 24 Calculus AB sample problems from their Advanced Placement booklet as items for the test (see Appendix A). Each student was interviewed and asked to "think aloud" while working on the

test items. Videotapes were made of each interview focusing primarily on the calculator and its use in the problem solving process. A one-hour time limit was enforced.

Coding of the Videotapes

Each student's videotape was coded for the reasoning processes used on each item. A sample of the coding form used appears in Appendix B and specifies the coding symbols used for all nineteen processes. The process scores are a frequency count of how many times each process was used by a student during work on each problem. A second coder, Mary Fulton, was hired to help with the coding of the videotapes and to determine an intercoder reliability coefficient for external validity purposes. Forty problems were coded independently by Mary Fulton and myself. Reliability coefficients of .63 or higher were obtained for the processes of reading, diagramming, manipulation, algorithm, deduction, working backwards, equations, and checking by retracing one's steps. The remaining processes were so seldom used by students that reliability coefficients could not be calculated but coding questions regarding them were handled on an individual basis.

Scoring of the Calculus Problems

The actual score a student received on an item is called his product score and ranged from 0 to 4. Since the AP Calculus AB test questions have a multiple choice answer format, a system of allocating partial credit on each item was devised to better discriminate between students' performances than by grading a problem just right or wrong. The key to the test (see Appendix A) shows how the partial credit was granted.

Analysis of the Data

Each item was analyzed for differences between experimental and control groups on process and product scores. Calculator use on each item was summarized. Because the sample size was so small (9 in the calculator group and 10 in the noncalculator group), statistical tests at the individual item level made no sense, rather the causes for what appeared to be large differences in either a process or product score on an item were identified by reviewing the videotapes for that particular item.

For all 24 items on the test, mean frequencies and standard deviations were calculated for each process to determine which processes were most often used by students in calculus problem solving and to determine the degree of variability of that use between students. The null hypothesis $H_o: \mu_c - \mu_{nc} = 0$ was tested against the alternative hypothesis $H_a: \mu_c - \mu_{nc} \neq 0$ to determine whether the mean product scores of students in the calculator and noncalculator groups were significantly different. Correlation coefficients were calculated between total process and product scores to determine which processes students were most successful with in solving calculus problems (see Appendix C).

Results of the Research Study

The results of the research study indicated that the use of calculators on AP calculus problems affected the testing of basic facts and reasoning processes at the individual problem level but basic concepts and principles were unaffected.

Students in the calculator group used their calculators to recall basic algebraic and trigonometric facts such as $e^{\circ} = 1$, $\sin \pi = 0$ and $\cos \frac{\pi}{2} = 0$. (see items 1, 19, and 20 of Appendix D.) Students in the noncalculator group were accustomed to recalling such facts from their calculator and when not allowed to use their calculators would frequently guess these values incorrectly. Several members of the noncalculator group got problems wrong due to incorrect recall of basic algebraic and trigonometric facts.

The use of calculators changed the strategies used to solve only a couple problems. In seven of the 24 problems, no one in the calculator group used their calculator in solving the problems and no one in the noncalculator group indicated that they would have liked to use a calculator. Because of the multiple choice format of the test, certain types of problems, (for example, see item 14 in Appendix D) could be solved by working backwards if you had a calculator that could not be solved by working backwards without a calculator. Problems where the domain of a function is an issue are particularily susceptible because calculators indicate an error if you attempt to find the value of a function at a point not in the domain of the function.

The use of exploratory manipulations with the calculator on the limit problem (see item 1, Appendix D) was more effective with the calculator group than the noncalculator group. Students with calculators could guess the answer to the limit problem by evaluating the function for small nonzero values of h but the noncalculator group could not obtain numerical guesses in this manner as easily.

The frequency of checking by retracing one's steps for the calculator group was approximately twice the frequency of checking by retracing one's steps for the noncalculator group. In particular, students in the calculator group were more inclined to check arithmetic calculations than students without calculators. Since checking by retracing one's steps correlated a positive .47 (see Appendix C) with product scores, this may be an important difference between testing calculus with and without the calculator.

The frequency of other processes used by students in the study either did not differ greatly between calculator and noncalculator groups or extant differences could not be linked to the calculator after reviewing the videotapes.

Despite the results reported above on the problem level, there was no significant difference between the mean product scores of the calculator (c) and noncalculator (nc) groups for the total test. An independent t value of -.20 was calculated and the null hypothesis H_o: $\mu_c - \mu_{nc} = 0$ was accepted at the .05 level of significance.

From the research study, it was apparent that the level of calculator expertise among MTU students limited their ability to take full advantage of their calculators as a tool in solving the AP calculus problems. MTU students used their calculators primarily as arithmetic machines and to recall basic algebraic and trigonometric facts. They were unaware of certain limitations of their calculators and did not know how to use the more powerful problem solving features of their calculators (e.g. storage, recall, and programming features).

The Teaching Experiment

During the fall quarter of 1980, I conducted a teaching experiment at MTU for the purpose of experimenting with calculator instruction and activities as they related to selected topics in calculus. I taught two consecutive classes of first quarter calculus. The first class served as the experimental group receiving calculator instruction as well as the regular calculus instruction. The second class served as the control group receiving only the regular calculus instruction.

Sample

These two calculus classes were composed primarily of engineering majors who owned their own pocket calculators. Of the 50 students in the experimental group, 40 students owned Texas Instrument (TI) calculators, 3 students owned Hewlett Packard (HP) calculators and the remaining 7 students owned other brands of calculators. Somewhat fewer than half the students owned calculators which were programmable.

Methodology

The results of the research study previously mentioned were used to help select calculus topics which would provide a contextual framework for the calculator instruction most needed by our students. The topics selected are shown below in Table 1.

Calculus Topics Used to Motivate Calculator Instruction

1. Composition of a function	Chain computations by use of the storage and recall keys or the parentheses keys.
2. A function and its inverse	Domain limitations of the y^x and $\frac{1}{x}$ keys
3. Limit of a function 4. ε,δ definition of a limit 5. Limit definition of a derivative	Writing and running simple programs to do iterative processes

Table 1

Each of the five lecture topics in Table 1 was introduced in a standard manner to both groups. In the experimental group, the last 15-20 minutes of class was spent in calculator instruction and calculator calculus activities. In the control group, additional sample problems were worked during this time.

Evaluative Procedures and Results

Students in the experimental group were also given a calculator calculus homework problem as an extension of their textbook assignment for each of the five calculus topics. Two of the five problems were AP Calculus AB sample problems used in the preceding research study. These two problems are given in Appendix E. A sample of student work for each problem is shown in italics. The mathematical facts, content, principles and reasoning processes used on these two problems by representatives of the teaching experiment were contrasted to those used by students in the research study. (See Appendix D).

In the first problem, the student evaluates the limit by writing a program for his calculator which will find values of the function for decreasing values of h. Such a brute force approach to limit problems is possible for a student who is good with his calculator. However, it decreases the chance of testing the intended content for this problem (i.e. the definition of the derivative of cos x when $x = \frac{\pi}{2}$).

In the second problem, the student checks the answers given to see which one works for a selected value of x. The calculator enables him to check answer A; whereas, without a calculator this would have been difficult. Therefore, students without a calculator would be less able to work backwards and use checking as processes for solving this problem. They would more likely have to resort to an algebraic algorithm for finding the inverse of f.

A comprehensive final examination was given to the experimental and control groups to compare their calculus achievement levels. An independent t-test ($t = .675$, D.F. = 88) revealed no significant difference ($p < .05$) between the mean examination scores for the experimental and control groups. This result implies that the 75-100 minutes of class time taken by the experimental group for calculator instruction and activities did not detract in any significant way from the calculus actually learned.

Thirty-eight students from the experimental group evaluated the calculator calculus component of the course using an attitudinal survey form which I devised. Most students felt that the calculator instruction was compatible with the calculus course and were enthusiastic about seeing the calculator instruction and activities continued on a supplemental basis in later terms of calculus. Twenty students claimed that the calculator calculus activities had helped them understand the calculus concepts better and three students reported that their new calculator skills had already proven useful in others courses. In general, the students regarded the teaching experiment as a success.

In summary, it appears to me that a more active involvement on the part of mathematics faculty is going to be needed to motivate students to learn about their calculators and help them gain expertise with their calculators.

The calculator's potential impact on the content validity of current AP calculus problems may be greater than we expect. Limit problems are solvable by evaluation of the function at limiting values of the independent variable. Programmable calculators are very useful in this respect. Domain and inverse function problems are solvable by working backwards and checking due to the interaction of the multiple choice format of the AP Calculus AB test and the quick evaluative powers of the calculator. Research studies addressing the question of the impact of the calculator on the content validity of AP calculus problems should be careful to evaluate the level of calculator expertise of the participants.

Appendix A

Test Items (the Key)

Subject Number KEY

Type of Calculator _____

Group _____
(experimental/control)

The Impact of the Calculator on the Reasoning Processes Used in Calculus Problem Solving

Instructions

This test consists of 24 multiple choice problems. Solve each problem using the space provided for scratchwork and put the answer in the blank to the left of the problem. Do all your thinking aloud so your reasoning processes can be followed. If you are in the experimental group, indicate whether or not you used your calculator to solve the problem in the blank to the right. If you are in the control group, indicate whether or not you would have liked to use the calculator to solve the problem in the blank to the right. On the next page there is a sample problem to try before you begin the test itself.

Notes: In this test, ln x denotes the natural logarithm of x (that is, logarithm to the base e). Unless otherwise specified, the domain of a function f is to be the set of all real numbers x for which f(x) is a real number.

Answer

Calculator Use (yes/no)

C ___ Sample Problem: If the graph of $y = 2^{-x} - 1$ is reflected in the x-axis, then an equation of the reflection is $y =$ _____

(A) $2^x - 1$ (B) $1 - 2^x$ (C) $1 - 2^{-x}$

(D) $\log_2(x+1)$ (E) $\log_2(1-x)$

reflection = $y = -(2^{-x} - 1) = 1 - 2^{-x}$

B ___ 1. What is $\lim_{h \to 0} \frac{\cos(\frac{\pi}{2} + h) - \cos\frac{\pi}{2}}{h}$? _____

(A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) $+\infty$

2pts a correct method. Defn. of a derivative $= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$

2pts correct answer as well. So, $\frac{d}{dx} \cos\frac{\pi}{2} = \lim_{h \to 0} \frac{\cos(\frac{\pi}{2}+h) - \cos\frac{\pi}{2}}{h} = -\sin\frac{\pi}{2} = -$

Alternate methods: try small values of h (in radians) / Partial Credit

B ___ 2. At which of the five points on the graph in the figure at the right are $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ both negative? L'Hospital's Rule _____

(A) A (B) B (C) C

(D) D (E) E

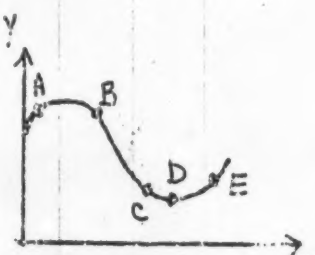
A. $f' > 0$ $f'' < 0$

B. $f' < 0$ $f'' < 0$

C. $f' < 0$ $f'' > 0$

D. $f' = 0$ $f'' > 0$

E. $f' > 0$ $f'' > 0$



1pts $m < 0$ correct interpretation of 1st derivative

2pts $f'' < 0$ means concave down

Partial Credit: notes about increasing/decreasing fctn and concave up/down

E ___ 3. If $y = x + \sin(xy)$, then $\frac{dy}{dx} =$ _____

(A) $1+\cos(xy)$ (B) $1+y\cos(xy)$ (C) $\frac{1}{1-\cos(xy)}$

(D) $\frac{1}{1-x\cos(xy)}$ (E) $\frac{1+y\cos(xy)}{1-x\cos(xy)}$

$y' = 1 + \cos(xy)(xy' + y)$

2 points $y' = 1 + xy'\cos xy + y\cos(xy)$ ← Partial Credit to here

$y' - xy'\cos(xy) = 1 + y\cos(xy)$

$y'(1 - x\cos(xy)) = 1 + y\cos(xy)$

$y' = \frac{1 + y\cos xy}{1 - x\cos xy}$

Calculator Use (yes/no)

C 4. Which of the following statements about the curve $y = x^4 - 2x^3$ is true?

(A) The curve has no relative extremum.

(B) The curve has one point of inflection and two relative extrema.

(C) The curve has two points of inflection and one relative extremum.

(D) The curve has two points of inflection and two relative extrema.

(E) The curve has two points of inflection and three relative extrema.

Partial Credit: $y' = 4x^3 - 6x^2$
$= 2x^2(2x-3)$
$x = 0, \ x = 3/2$
1 (pt) for critical pts

Determine max/min
$y'' = 12x^2 - 12x$
$y''(0) = 0$
$y''(3/2) = 12(\frac{9}{4}) - 12(\frac{3}{2})$
$= 9 > 0$
so $3/2$ is a min. (1 pt)

Determine inflection
$12x^2 - 12x > 0$
$12x(x-1) > 0$
$x>0 \quad x>1$
$x = 0, 1$ are inflection pts (2 pts)

E 5. If h is the inverse function of f and if $f(x) = \frac{1}{x}$, then $h(x) =$

(A) $\ln x$ (B) $-\frac{1}{x^2}$ (C) $-\frac{1}{x}$ (D) x (E) $\frac{1}{x}$

Algorithms
2 pts for method
$y = \frac{1}{x}$
$x = \frac{1}{y}$
$y = \frac{1}{x}$
2 pts answer $h(x) = \frac{1}{x}$

$f[h(x)] = x$ and $h[f(x)] = x$ relationship
2 pts for method

A. $f(\ln x) = \frac{1}{\ln x}$
B. $f(-\frac{1}{x^2}) = -x^2$
C. $f(-\frac{1}{x}) = -x$
D. $f(x) = \frac{1}{x}$
*E. $f(\frac{1}{x}) = \frac{1}{\frac{1}{x}} = x$
2 pts for answer

C 6. If $f(x) = x^5 - x^4 + 2x - 1$ and if r is the only real number such that $f(r) = 0$, then r is between

(A) -2 and -1 (B) -1 and 0 (C) 0 and 1
(D) 1 and 2 (E) 2 and 3

Check for sign change
A) $f(-2) = -32 - 16 - 4 - 1 = -53$
B) $f(-1) = -1 - 1 - 2 - 1 = -5$, $f(0) = -1$
C) $f(1) = 1 - 1 + 2 - 1 = 3$
Answer is C by Intermediate Value Theorem.

2 pts for a method that could give a correct solution
2 pts for correct answer

Calculator Use (yes/no)

C 7. A smooth curve has the property that for all x the value of its slope at 2x is twice the value of its slope at x. The slope at 0 is

(A) not defined
(B) negative
(C) zero
(D) positive
(E) not determinable from the information given.

$f'(2x) = 2f'(x)$
$f'(2 \cdot 0) = 2f'(0)$
$f'(0) = 2f'(0)$
So $f'(0)$ must be 0

Partial Credit { (+1) graph with slope doubling
(+2) a little more intelligent work
(+1) correct answer, as well.

B 8. Let $f(x) = \frac{1}{k}\cos(kx)$. For what value of k does f have period 3?

(A) $\frac{2}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{3\pi}{2}$ (D) 6 (E) 6π

Period $\frac{2\pi}{b}$ for $y = a\cos(bx+c)$

$3 = \frac{2\pi}{k}$

$k = \frac{2\pi}{3}$

Partial Credit: { 2 pts correct method
2 pts. correct answer, as well.

D 9. $\int (x-1)\sqrt{x}\,dx =$

(A) $\frac{3}{2}\sqrt{x} - \frac{1}{\sqrt{x}} + C$ (B) $\frac{2}{3}x^{3/2} + \frac{1}{2}x^{1/2} + C$

(C) $\frac{1}{2}x^2 - x + C$ (D) $\frac{2}{5}x^{5/2} - \frac{2}{3}x^{3/2} + C$

(E) $\frac{1}{2}x^2 + 2x^{3/2} - x + C$

$\int (x-1)x^{1/2}\,dx = \int (x^{3/2} - x^{1/2})\,dx$ ← +2 pts credit

$= \frac{2}{5}x^{5/2} - \frac{2}{3}x^{3/2} + C$ ← 2 pts. for correct integration & answer

Answer

Calculator Use (yes/no)

B 10. What is $\lim_{x\to\infty} \frac{x^2 - 4}{2 + x - 4x^2}$?

(A) -2 (B) $-\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 1

(E) The limit does not exist.

2 pts. for finding a correct method. factoring fails to work
brute force calculator methods works (eg. let x = 1
divide numerator & denominator by x^2 works

$$\lim_{x\to\infty} \frac{\frac{x^2}{x^2} - \frac{4}{x^2}}{\frac{2}{x^2} + \frac{x}{x^2} - \frac{4x^2}{x^2}} = \lim_{x\to\infty} \frac{1 - \frac{4}{x^2}}{0 + 0 - 4} = -\frac{1}{4}$$

2 pts for correct answer.

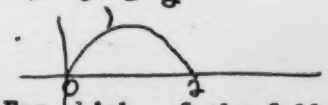
B 11. The area of the region in the first quadrant between the graph of $y = x\sqrt{4 - x^2}$ and the X-axis is

(A) $\frac{2}{3}\sqrt{2}$ (B) $\frac{3}{3}$ (C) $2\sqrt{2}$ (D) $2\sqrt{3}$ (E) $\frac{16}{3}$

(1pt) Find x-intercepts

$0 = x\sqrt{4 - x^2}$

$x = 0, \pm 2$



(1pt) $\int_0^2 x(4 - x^2)^{1/2} dx$

let $u = 4 - x^2$ (1pt)

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$\int_4^0 -\frac{1}{2} u^{1/2} du$

$\frac{1}{2}\left(\frac{2}{3}u^{3/2}\Big|_0^4\right)$

$\frac{1}{3}(8) = \frac{8}{3}$ (1pt)

D 12. For which of the following functions does the property $\frac{d^3y}{dx^3} = \frac{dy}{dx}$ hold?

I. $y = e^x$ $y' = e^x$ $y'' = e^x$ $y''' = e^x$

II. $y = e^{-x}$ $y' = -e^{-x}$ $y'' = e^{-x}$ $y''' = -e^{-x}$

III. $y = \sin x$ $y' = \cos x$ $y'' = -\sin x$ $y''' = -\cos x$

2 pts for correct derivatives

(A) I only (B) II only (C) III only

(D) I and II (E) II and III

(2pts) for correct answer.

Calculator Use (Yes/No)

C 13. Let $a < c < b$ and let f be differentiable on $[a,b]$. Which of the following is NOT necessarily true?

T (A) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

T (B) There exists d in $[a,b]$ such that $f'(d) = \frac{f(b) - f(a)}{b - a}$ mean value theorem

(C) $\int_a^b f(x)dx \geq 0$

T (D) $\lim_{x \to c} f(x) = f(c)$ differentiability $\Rightarrow$ continuity

T (E) If k is a real number, then $\int_a^b kf(x)dx = k\int_a^b f(x)dx$.

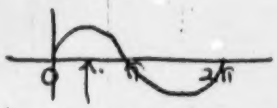
2pts. for eliminating at least 3 choices
2pts. for correct answer.

A 14. The function $f(x) = \ln(\sin x)$ is defined for all x in which of the following intervals?

OK (A) $0 < x < \pi$ no (B) $0 \leq x \leq \pi$

no (C) $\frac{3\pi}{2} < x < \frac{5\pi}{2}$ no (D) $\frac{3\pi}{2} \leq x \leq \frac{5\pi}{2}$

no (E) $\frac{3\pi}{2} < x < 2\pi$



(2pts) for valuating the function at pts of intervals
(2pts) for correct answer

$\sin x > 0$ for $0 < x < \pi$
domain of ln is positive reals only.

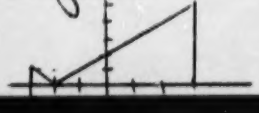
D 15. $\int_{-3}^{3} |x+2|\,dx =$

(A) 0 (B) 9 (C) 12 (D) 13 (E) 14

1 pt. for splitting up integral: $\int_{-3}^{-2} -(x+2)dx + \int_{-2}^{3} (x+2)dx =$
1 pt. for getting limits on integrals correct
1 pt. for integration
1 pt. for algebra + answer $-\left(\frac{x^2}{2} + 2x\Big|_{-3}^{-2}\right) + \left(\frac{x^2}{2} + 2x\Big|_{-2}^{3}\right) =$

$\frac{5}{2} - 2 + \frac{5}{2} + 10 = 13$

Graphical Solution:



2pts correct graph

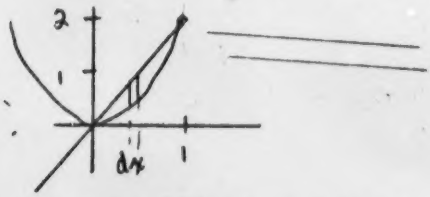
Calculator Use (yes/no)

B ___ 16. The volume generated by revolving about the Y-axis the region enclosed by the graphs of $y = 2x$ and $y = 2x^2$, for $0 \le x \le 1$, is ___

(A) $\pi \int_0^1 (2x - 2x^2)^2 dx$ (B) $\pi \int_0^1 (4x^2 - 4x^4) dx$

(C) $2\pi \int_0^1 x(2x - 2x^2) dx$ (D) $\pi \int_0^2 \left(\sqrt{\frac{y}{2}} - \frac{y}{2}\right)^2 dy$

(E) $\pi \int_0^2 \left(\frac{y}{2} - \frac{y^2}{4}\right) dy$



2pts correct pts of intersection for limits of integral

2pts correct solution

D ___ 17. Let f be defined as follows, where $a \neq 0$ ___

$$f(x) = \begin{cases} \dfrac{x^2 - a^2}{x - a}, & \text{for } x \neq a, \\ 0, & \text{for } x = a. \end{cases}$$

$\frac{(x-a)(x+a)}{x-a}$

Which of the following are true about f?

T I. $\lim_{x \to a} f(x)$ exists. 1pt

T II. $f(a)$ exists. 1pt

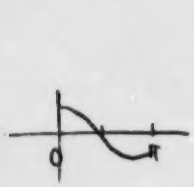
F III. $f(x)$ is continuous at $x = a$. 1 pt

(A) None (B) I only (C) II only (D) I and II only.
(E) I, II, and III

1 pt. conclusion

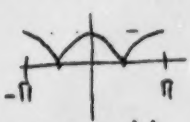
E ___ 18. Which of the following definite integrals is NOT equal to 0? ___

(A) $\int_{-\pi}^{\pi} \sin^3 x \, dx = 0$ (B) $\int_{-\pi}^{\pi} x^2 \sin x \, dx = 0$



(C) $\int_0^{\pi} \cos x \, dx = 0$ (D) $\int_{-\pi}^{\pi} \cos^3 x \, dx = 0$

(E) $\int_{-\pi}^{\pi} \cos^2 x \, dx \neq 0$



17

2pts if eliminates correctly 2 choices

2pts correct answer

D

19. The acceleration at time t of a [illegible] moving on the x-axis is $4\pi\cos t$. If the velocity is 0 at $t = \pi$, what is the average velocity of the particle over the interval $0 \le t \le \pi$?

(A) 0 (B) $\frac{4}{\pi}$ (C) 4 (D) 8 (E) 2π

$a = 4\pi\cos t$

$v = 4\pi\sin t + C$, $v(\pi) = 0$

so $0 = 0 + C$, so $C = 0$

(2pts) $v = 4\pi\sin t$

average velocity $= \frac{v(\pi) - v(0)}{\pi - 0}$

$= \frac{-4\pi(-\pi) + (-(-4\pi\cdot 1))}{\pi}$

$= 8$ (2pts) for answer

A

20. $\int_0^1 xe^{-x}\,dx =$

(A) $\frac{e-2}{e}$ (B) $\frac{2-e}{e}$ (C) $\frac{e-2}{e}$ (D) $\frac{e}{e-2}$ (E) $\frac{e}{2-e}$

(1pt) for recognizing as an integration by parts problem & correctly choosing u and dv

$u = x \quad dv = e^{-x}dx$

$du = dx \quad v = -e^{-x}$

(1pt) $\int_0^1 xe^{-x}dx = -xe^{-x}\Big|_0^1 + \int_0^1 e^{-x}dx$

$= -xe^{-x}\Big|_0^1 - e^{-x}\Big|_0^1$

$= -e^{-1} - e^{-1} - (0-1)$

$= -2e^{-1} + 1$ ← (1pt)

$= -\frac{2}{e} + \frac{e}{e} = \frac{-2+e}{e}$ (1pt)

A

21. Let $f(x) = x^3 + x$. If h is the inverse function of f, then $h'(2) =$

(A) $\frac{1}{13}$ (B) $\frac{1}{4}$ (C) 1 (D) 4 (E) 13

(2pts) $h'(x) = \frac{1}{f'(x)} = \frac{1}{3x^2+1}$

(2pts) $h'(2) = \frac{1}{3\cdot 2^2+1} = \frac{1}{13}$

Alternate method:

1pt find inverse function

2pts. take derivative of inverse fctn.

1pt. correct answer.

A

22. $\int \cos^2 x \sin x\,dx =$

(A) $-\frac{\cos^3 x}{3} + C$ (B) $-\frac{\cos^3 x + \ln^2 x}{6} + C$

(C) $\frac{\sin^2 x}{2} + C$ (D) $\frac{\cos^3 x}{3} + C$

(E) $\frac{\cos^3 x \sin^2 x}{6} + C$

1pt. for recognizing as a substitution problem

1pt $u = \cos x$

$du = -\sin x\,dx$

$\int \cos^2 x \cdot \sin x\,dx = \int -u^2\,du$

$= -\frac{u^3}{3} + C = -\frac{\cos^3 x}{3} + C$ (2pts)

Answer

B

23. If r is positive and increasing, for what value of r is the rate of increase of r^3 twelve times that of r?

(A) $\sqrt[3]{4}$ (B) 2 (C) $\sqrt[3]{12}$ (D) $2\sqrt{3}$ (E) 6

$\frac{d}{dr} r^3 = 12 \frac{d}{dr} r$ (2pts)

$3r^2 = 12$

$r^2 = 4$

$r = \pm 2$ but only $r = 2$ is positive. (2pts answer)

A

24. If f is continuous for all x, which of the following integrals necessarily have the same value?

I. $\int_a^b f(x)dx = F(b) - F(a)$ 1pt

II. $\int_0^{b-a} f(x + a)dx = F(b - a + a) - F(a)$ 1pt.

III. $\int_{a+c}^{b+c} f(x + c)dx = F(b + c + c) - F(a + c + c)$ 1pt

(A) I and II only (B) I and III only 1pt. correct answer

(C) II and III only (D) I, II, and III

(E) No two necessarily have the same value.

Appendix B

Coding Form

CODING FORM FOR PROBLEM SOLVING PROCESSES

SUBJECT NUMBER # 3 CODER MF TAPE READINGS 656-665

ITEM NUMBER 24 DATE 9 16 80 ITEM TIME _____ minutes

PROCESS SEQUENCE: R, AL.

PROBLEM SOLVING PROCESSES BY CATEGORY:

PREPARATION (Coding Symbol)

1 Reads the problem to try to understand it (R)

_____ Represents the problem with a diagram (DI)

_____ Performs exploratory manipulation (M)

_____ Groups the data by construction of a chart (CT)

_____ Changes the conditions of the problem to be more specialized (S)

_____ Changes the conditions of the problem to be more general (G)

RECALL

_____ Uses the results or methods of a previously worked problem --analogy (A)

1 Uses an algorithm committed to memory (AL)

PRODUCTION

_____ Uses trial and error (TE)

_____ Uses successive approximations (SA)

_____ Uses deduction (D)

_____ Uses induction (I)

_____ Uses equation(s) (E)

_____ Uses reduction-combination (RC)

_____ Works backwards (WB)

EVALUATION

_____ Checks that solution satisfies original equation or conditions of problem (CI)

_____ Checks solution by retracing steps (CII)

_____ Checks that solution is reasonable or realistic (CIII)

_____ Derives the solution by another method (CIV)

CODER'S ADDITIONAL COMMENTS: _____

Appendix C

Means, Standard Deviations and Correlation with Product Scores for Each Process across all 24 items

PROCESS	MEAN FREQUENCY ON ALL 24 ITEMS	STANDARD DEVIATION	CORRELATION OF PROCESS AND PRODUCT SCORES
READING	26.32	5.08	.21
DIAGRAMMING	3.85	1.90	.07
MANIPULATION	.68	.95	-.13
ANALOGY	.16	.37	-.15
ALGORITHM	19.00	4.46	.75
DEDUCTION	1.89	1.73	.08
WORKING BACKWARDS	4.74	3.19	-.25
SUCCESSIVE APPROXIMATIONS	.11	.32	.14
EQUATIONS	.95	1.13	.31
TRIAL & ERROR	.16	.69	.11
CHECKING (SATISFIES CONDITIONS)	.16	.50	-.15
CHECKING (RETRACES STEPS)	2.11	2.11	.47
CHECKING (REASONABLE)	.05	.23	.71
CHECKING (ALTERNATE METHOD)	.16	.50	.68

Appendix D

Selected Problems Analyzed Individually from the Research Study

ITEM 1: WHAT IS $\lim_{H \to 0} \frac{\cos(\frac{\pi}{2} + H) - \cos\frac{\pi}{2}}{H}$

_____ (A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) $+\infty$

TABLE 1

PROCESS	FREQUENCY OF STUDENT USE OF THE PROCESS VARIABLES EXPERIMENTAL (N = 8)	CONTROL (N = 9)
READING	8	9
DIAGRAMMING	2	0
MANIPULATION	4	3
ANALOGY	0	2
ALGORITHM	7	4
DEDUCTION	1	2
WORKING BACKWARDS	2	0

N = 17

CALCULATOR USE: FIVE OF THE 8 STUDENTS IN THE EXPERIMENTAL GROUP USED THE CALCULATOR ON THIS ITEM. TWO STUDENTS PLUGGED IN A NUMBER CLOSE TO 0 FOR H, BUT BOTH GOT THE LIMIT WRONG AS THEY DID NOT USE CALCULATOR CORRECTLY. TWO STUDENTS USED THEIR CALCULATOR TO RECALL THAT $\cos\frac{\pi}{2} = 0$

GENERAL COMMENTS: ONLY TWO STUDENTS RECOGNIZED THIS PROBLEM AS A QUESTION REGARDING THE DEFINITION OF THE DERIVATIVE OF $\cos\theta$ AT THE POINT $\theta = \frac{\pi}{2}$.

SCORES ON ITEM 1. (0-4)

$\bar{x} = 1.14$ $s_x = (1.57)$ EXPERIMENTAL

$\bar{y} = .89$ $s_y = (1.76)$ CONTROL

ITEM 5: IF H IS THE INVERSE FUNCTION OF F AND IF $F(X) = \frac{1}{X}$, THEN $H(X)$ =

(A) LN X (B) $-\frac{1}{X^2}$ (C) $-\frac{1}{X}$ (D) X (E) $\frac{1}{X}$

TABLE 5

PROCESS	NUMBER OF STUDENTS USING THE PROCESS VARIABLES	
	EXPERIMENTAL (N = 9)	CONTROL (N = 10)
READING	14	13
DIAGRAMMING	0	1
MANIPULATION	1	0
ALGORITHM	7	9
DEDUCTION	1	1
WORKING BACKWARDS	2	0
CHECKING (RETRACES STEPS)	1	0

N = 19

CALCULATOR USE: STUDENTS DID NOT USE THE CALCULATOR ON THIS PROBLEM. IT COULD HAVE BEEN USED TO CHECK WHETHER $H[G(X)] = X$ AND $G[H(X)] = X$ FOR THE CHOICES GIVEN.

SCORES ON ITEM 5: (0 - 4)

$\bar{X} = 2.00$ $S_X = 2.00$ EXPERIMENTAL

$\bar{Y} = 2.00$ $S_Y = 2.11$ CONTROL

ITEM 14: THE FUNCTION $F(X) = LN(SIN\ X)$ IS DEFINED FOR ALL X IN WHICH OF THE FOLLOWING INTERVALS?

(A) $0 < X < \pi$

(B) $0 \leq X \leq \pi$

(C) $\frac{3\pi}{2} < X < \frac{5\pi}{2}$

(D) $\frac{3\pi}{2} \leq X \leq \frac{5\pi}{2}$

(E) $\frac{3\pi}{2} < X < 2\pi$

TABLE 14

PROCESS	FREQUENCY OF STUDENT USE OF THE PROCESS VARIABLES	
	EXPERIMENTAL (N = 9)	CONTROL (N = 10)
READING	14	12
DIAGRAMMING	4	1
ALGORITHM	2	4
DEDUCTION	1	0
WORKING BACKWARDS	9	1 *
TRIAL AND ERROR	1	0
CHECKING (RETRACES STEPS)	1	0

CALCULATOR USE: TO USE THE CALCULATOR TO DETERMINE WHERE LN SIN X WAS DEFINED. STUDENTS WOULD SUBSTITUTE A VALUE(S) OF X FROM THE INTERVALS GIVEN IN MULTIPLE CHOICE ANSWERS AND USE THEIR CALCULATORS TO SEE IF F(X) WAS DEFINED OR IF THEIR CALCULATOR INDICATED AN ERROR.

* WORKING BACKWARDS WAS A POPULAR PROCESS FOR THIS PROBLEM IF YOU HAD A CALCULATOR BUT SELDOM USED BY STUDENTS IN THE CONTROL GROUP.

SCORES ON ITEM 14: (0 - 4)

$\bar{x} = 1.78$ $s_x = 1.86$ EXPERIMENTAL GROUP

$\bar{y} = 1.90$ $s_y = 2.02$ CONTROL GROUP

ITEM 19: THE ACCELERATION AT TIME T OF A PARTICLE MOVING ON THE X-AXIS IS $4\pi \cos t$. IF THE VELOCITY IS 0 AT $T = 0$, WHAT IS THE AVERAGE VELOCITY OF THE PARTICLE OVER THE INTERVAL $0 \leq T \leq \pi$?

(A) 0 (B) $\frac{4}{\pi}$ (C) 4 (D) 8 (E) 8π

TABLE 9

PROCESS	FREQUENCY OF STUDENT USE OF THE PROCESS VARIABLES EXPERIMENTAL (N = 7)	CONTROL (N = 8)
READING	9	11
ALGORITHM	6	6
DEDUCTION	1	0
WORKING BACKWARDS	1	1
EQUATIONS	1	0
CHECKING (RETRACES STEPS)	0	1

CALCULATOR USE: TWO STUDENTS IN THE EXPERIMENTAL GROUP USED THEIR CALCULATOR TO RECALL BASIC TRIG FACTS SUCH AS $\sin \pi = 0$ AND $\sin 0 = 0$. TWO STUDENTS IN THE CONTROL GROUP INDICATED THEY WOULD HAVE LIKED TO USE THEIR CALCULATORS FOR THE SAME PURPOSE.

SCORES ON ITEM 19: (0-4)

$\bar{x} = 1.00$ $s_x = 1.15$ EXPERIMENTAL

$\bar{y} = 1.00$ $s_y = 1.51$ CONTROL

ITEM 20: $\int_0^1 XE^{-X}DX =$

_____ (A) $\frac{E-2}{E}$ (B) $\frac{2-E}{E}$ (C) $\frac{E+2}{E}$ (D) $\frac{E}{E-2}$ (E) $\frac{E}{2-E}$

TABLE 20

PROCESS	FREQUENCY OF STUDENT USE OF THE PROCESS VARIABLE EXPERIMENTAL (N = 7)	CONTROL (N = 9)
READING	7	9
ALGORITHM	8	12
WORKING BACKWARDS	1	1
CHECKING (RETRACES STEPS)	6	6

CALCULATOR USE: ONE STUDENT IN THE EXPERIMENTAL GROUP USED HER CALCULATOR TO EVALUATE E^0 AND E^{-1}, ALTHOUGH THE ANSWER WAS NOT GIVEN IN DECIMAL FORM.

SCORES ON ITEM 20: (0-4)

$\bar{x} = 2.57$ $s_x = 1.62$ EXPERIMENTAL

$\bar{y} = 1.78$ $s_y = 1.48$ CONTROL

Appendix E

Sample Calculator Calculus Homework Problems

The Teaching Experiment

Problem 1 in Appendix D

MA 150 Section 10 **Name** *Student*

Dr. Gimmestad **Type of Calculator** *TI 58C*

General Directions: Please turn this problem in at the end of class on Wednesday, September 24.

Problem: Find $\lim_{h\to 0} f(h) = \lim_{h\to 0} \dfrac{\cos(\frac{\pi}{2} + h) - \cos\frac{\pi}{2}}{h}$

If you have a programmable calculator, try writing a program to evaluate f(h) as values of h vary. If you do not have a programmable calculator, you must repeat all the steps in the evaluation of f(h) for each new value of h. Indicate your keystrokes below.

KEYSTROKES:

PROGRAM |LRN| |STO| |1| |2nd| |RAD| |(| |(| |(| |2nd| |π| |÷| |2| |+|

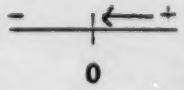
|RCL| |1| |)| |2nd| |COS| |)| |-| |(| |2nd| |π| |÷| |2| |)|

|2nd| |COS| |)| |÷| |RCL| |1| |=| |R/S| |RST| |LRN| |RST|

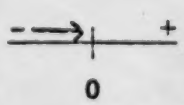
RUNNING IT: |h| |R/S|

Fill in the following tables

h	.3	.2	.1	.05	.005
f(h)	-.9850	-.9933	-.9983	-.9995	-.9999

Note: $h\to 0^+$ 

h	-.3	-.2	-.1	-.05	-.005
f(h)	-.9850	-.9933	-.9983	-.9995	-.9999

Note: $h\to 0^-$ 

Answer: $\lim_{h\to 0} f(h) =$ *-1*

Please estimate the amount of time it took you to do this assignment *15* **min.**

Problem 5 in Appendix D

MA 150 Section 10 Name Student

Dr. Gimmestad Type of Calculator TI 57

General Directions: Please turn this problem in at the end of class on Wednesday, Sept. 31. Please indicate your keystrokes when you use your calculator.

Problem: If h is the inverse function of f and if $f(x) = \frac{1}{x}$, then $h(x) =$

E (A) $\ln x$ (B) $-\frac{1}{x^2}$ (C) $-\frac{1}{x}$ (D) X (E) $\frac{1}{x}$

Student Work

Use your calculator at $x = 5$ to check if $h[f(5)] = 5$ for each possible $h(x)$. Note that $f(5) = \frac{1}{5}$.

A. $h[f(5)] = \ln \frac{1}{5} = -1.6094379 \neq 5$

Keystrokes: |5| |1/x| |ln x|

B. $h[f(5)] = -\frac{1}{(\frac{1}{5})^2} = -25 \neq 5$

C. $h[f(5)] = -\frac{1}{\frac{1}{5}} = -5 \neq 5$

D. $h[f(5)] = \frac{1}{5} \neq 5$

E. $h[f(5)] = \frac{1}{\frac{1}{5}} = 5$

Please estimate the amount of time it took you to do this assignment 10 min.